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Application of the truncated gaussian to the inelastic pp charged multiplicity distribution

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Abstract. It is shown that inelastic pp charged particle multiplicity distributions from 13 to 300 GeV/c can be successfully described by means of a truncated gaussian. An interpretation of this phenomenology is made through a parton-like model. Extrapolation of this fit to higher energies implies that no double-bump structure will occur, contrary to the currently fashionable two-component picture of particle production. The extension to other processes is briefly discussed.

1. Introduction

The Poisson distribution has been the most popular candidate for describing the inelastic charged particle topological cross sections (σ_n) for some time. The low-energy data were well fitted by Wang (1969a, b) using a Poisson distribution in charged pairs. Added support was given by the fact that several theoretical models could be constructed which predicted Poisson-like distributions (Chew and Pignotti 1968, Rotelli 1969). However, recent data at higher energies ($p_{lab} = 13$ GeV/c, Smith 1971; $p_{lab} = 19$ GeV/c, Bøggild *et al* 1971; $p_{lab} = 50$ and 69 GeV/c, Soviet–French collaboration 1971; $p_{lab} = 102$ GeV/c, Chapman *et al* 1972, $p_{lab} = 205$ GeV/c, Charlton *et al* 1972; $p_{lab} = 303$ GeV/c, Dao *et al* 1972) have shown that the simple Poisson distribution will not explain the production cross sections over the whole range of energy now available although, by a judicious choice of variable, good fits may be obtained even at (isolated) higher energies ($p_{lab} = 50$ GeV/c, Soviet–French collaboration 1972). The two main reasons why a Poisson is seen to be inadequate are (see Jacob 1972): (i) the second correlation function f_2 is not zero as predicted by Poisson, but increases with energy; (ii) the peak of each experimental distribution is substantially lower than its mean value even at the highest energies; a feature clearly shown in figure 9 of Jacob (1972).

Recently, to overcome these failures, several different authors have proposed the following scheme (Fiałkowski 1972, Van Hove 1973, Harari and Rabinovici 1973, Frazer *et al* 1973, Fiałkowski and Miettinen 1973). They assume that there are two types of production processes: diffraction, which is assumed to contribute large cross sections only for low-multiplicity final states, and a multiperipheral or pionization process, for which a favourite description is the Poisson. This succeeds to the extent that it can be tested, though its distinguishing feature is the ultimate development of a double-peaked structure. However, the inelastic data at present show no hint of such behaviour.

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In this paper we propose an alternative solution. We assume that the data asymptotically retain their single-bump structure and we allow for a non-Poisson dispersion by employing a gaussian or normal distribution. This is not a new idea and some of the advantages have already been discussed by Kaiser (1972). However, we take note of the fact that in translating the continuous distribution to the discrete experimental data points, Kaiser concluded, on the basis of low-energy data, that the elastic cross section must also be included for a satisfactory fit to be obtained. This prescription fails when applied to the new NAL data at 100–300 GeV/c. Our alternative prescription will ascribe successfully a truncated gaussian to the best inelastic data from 13–300 GeV/c.

As a plausibility argument we invoke a parton-like model in the context of which the continuous distribution may be understood and the origin of the gaussian justified on the basis of the mathematical law of large numbers applied to the partons.

The two-parameter nature of the gaussian will automatically accommodate point (i) made above, and point (ii) is found to follow from the finite range of the experimental distribution, in particular the lower limit of $n_{\rm ch} = 2$ in the proton–proton scattering. (The effect of the upper cut-off imposed by the finite masses of the hadrons is negligible.) This truncation plays a fundamental role in our fits and interpretation. In § 2 we describe the details of the phenomenology. In § 3 we present the naive model employed to interpret our fits and discuss its extension to other processes; § 4 concludes with a discussion of our analysis and the possible results this fit infers for higher energies.

2. Phenomenology

The normalized gaussian is given by

$$\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-a)^2}{2\sigma^2}\right), \tag{1}$$

where x ranges continuously from $-\infty$ to $+\infty$, and a and σ^2 are the average value of and dispersion in the variable x. We shall interpret x as $n_{\rm ch}$ and, since pp scattering always yields $n_{\rm ch} \geqslant 2$, we consider the truncated gaussian in which x runs continuously from 2 to $N_{\max} \simeq \infty$. The truncation means that a and σ^2 no longer correspond to $\langle x\rangle$ and $\langle (x-\langle x\rangle)^2\rangle$ respectively. Instead,

$$\langle x\rangle = a+\left(\frac{2}{\pi}\right)^{1/2}\frac{\sigma\exp\{-(2-a)^2/2\sigma^2\}}{1+\Phi((a-2)/\sqrt{2}\sigma)} \tag{2}$$

and

$$\langle x^2\rangle = a\langle x\rangle+\sigma^2+2\left(\frac{2}{\pi}\right)^{1/2}\frac{\sigma\exp\{-(2-a)^2/2\sigma^2\}}{1+\Phi((a-2)/\sqrt{2}\sigma)} \tag{3}$$

where

$$\Phi(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}\,dt. \tag{4}$$

The normalized truncated gaussian then becomes

$$P(x) = \left(\frac{2}{\pi}\right)^{1/2}\frac{1}{\sigma\{1+\Phi((a-2)/\sqrt{2}\sigma)\}}\exp\{-(x-a)^2/2\sigma^2\}. \tag{5}$$

It is also noted that

$$\langle x^2\rangle - \langle x\rangle(a+2)+2a = \sigma^2. \tag{6}$$

Now we must decide how to convert this continuous curve into the discrete integer values obtained experimentally. We assume that the probability of producing m charged particles for $m \geqslant 4$ (m even) is given by integrating $P(x)$ between $m-1$ and $m+1$; that is,

$$\frac{\sigma_m}{\sigma_{\text{inel}}} = \int_{m-1}^{m+1} P(x)\,\mathrm{d}x. \tag{7}$$

However, for the minimum charged multiplicity, that is, $n_{\text{ch}} = 2$, we are forced by the lower cut-off to define σ_2 as

$$\frac{\sigma_2}{\sigma_{\text{inel}}} = \int_2^3 P(x)\,\mathrm{d}x. \tag{8}$$

An interpretation of this procedure will be given in the next section. In fitting, we take as input $\langle n_{\text{ch}}\rangle$ and $\langle n_{\text{ch}}^2\rangle$ and use equations (6) and (2) to fix approximately the values of a and σ. These are then varied slightly until we obtain a best χ^2 fit. In table 1 we list the

Table 1. Results of fits to pp data

p_{lab} (GeV/c)	a	σ	a/σ	χ^2/N
12·88	1·5	2·33	0·635	5·5/7†
19	1·75	2·81	0·624	11·4/7
50	3·0	3·9	0·77	9·6/8
69	3·5	4·1	0·85	9·1/9
102	4·5	4·36	1·02	5·5/9
205	5·5	5·23	1·05	9·5/11
303	7·0	5·64	1·25	13·4/13

† σ_2 mode not included since not quoted in Smith (1971).

values of a and σ together with the χ^2 obtained. The 'anomalous' weighting of σ_2 distinguishes our phenomenology from that of Kaiser and explains why he was led to include the elastic data in order to reconcile theory with experiment. It is interesting to emphasize in passing that, unlike the Poisson distribution, the gaussian is capable of including diffractive and, in particular, elastic contributions *if* asymptotically $a \sim \sigma \sim \ln s$, in which case

$$\sigma_m \underset{s\to\infty}{\sim} \frac{1}{\ln s} \qquad \text{for all } m. \tag{9}$$

However, as we have already said, the high-energy data are in disagreement with Kaiser's version, while we are able to fit all the best data available. Indeed it is quite startling that for the lowest energies fitted (13, 19 GeV/c) the underlying gaussian peaks below 2, and hence outside the physical range. The experimental peak then occurs only because of the anomalous weighting of σ_2. It may be argued that, since we have excluded specifically the elastic cross section as is conventionally done, we should, on aesthetic grounds, also exclude the diffractive part of the inelastic cross sections. However, some theoretical estimates (Rotelli 1971) and the recent rate of fall-off of σ_4 at NAL energies

(Jacob 1972) suggest that these contributions are small. On this question there is a strong divergence of opinion.

As an example, we show our fits to the 19 and 303 GeV/c data in figure 1.

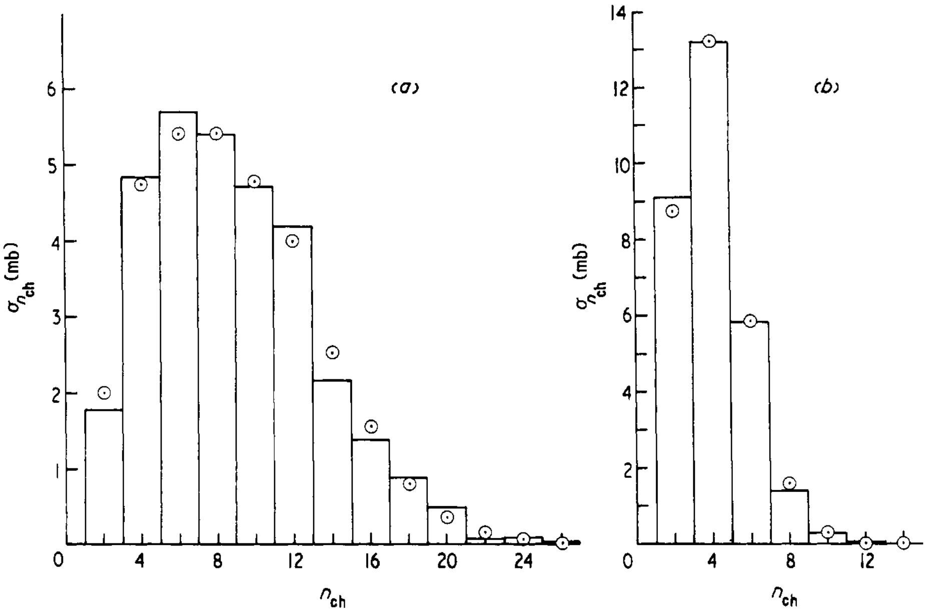

Figure 1. Our fits to the data at (a) p_{lab} = 303 GeV/c and (b) 19 GeV/c. ⊙ Theoretical values.

3. A naive model

The most direct, yet most revolutionary, way of interpreting the phenomenology presented in the previous section is to ascribe a physical meaning to the underlying continuous curve for charge distribution, that is, we postulate the existence of large numbers of constituent particles (partons) with small if not infinitesimal fractional charges. To be more specific, consider the following *simplified* version. Every charged elementary particle is constructed from, say, N partons each with charge $1/N$th of that of the elementary particle. Neutrals would consist of N neutral partons. In the collision of two protons a number of partons interact and create additional partons, subject of course to charge conservation (we ignore, for simplicity, the question of baryon number). At this stage we have a system with, in general, fractional charge multiplicity. The partons must now recombine by final state interactions to produce the observed *physical* final state. It is in this second stage that we envisage the transition back from fractional to integer charges. Our phenomenological procedure (§ 2) is equivalent to assuming that the underlying charge distribution is gaussian and that the final state interactions will predominantly convert the fractional virtual charge multiplicity to the nearest integer values. This model, of course, automatically explains the lower cut-off. It also suggests that the gaussian form is due to the application of the mathematical law of numbers, which, at the parton level, is valid in the above model even at $n_{ch} = 2$. This model can, of course, be made considerably more complicated and sophisticated. In fact we should like to introduce the concept of leading partons so that, even in π^-p interactions, the *majority* of the incoming partons pass straight through without interactions. Our motivation for making this suggestion is that a cut-off at $n_{ch} = 2$ for π^-p seems necessary to enable us to fit the existing limited data. The $n_{ch} = 0$ mode is exceedingly small and we interpret this as the low probability for the majority of partons to interact (strictly speaking, the effective phenomenological cut-off would then be at a value slightly less than 2).

Table 2 shows our fits to π^-p and π^-n at $p_{lab} = 40$ GeV/c (Anson *et al* 1970). For π^-p we neglect the $n_{ch} = 0$ mode and impose a cut-off at $n_{ch} = 2$ as justified above. For π^-n no such problem arises and the cut-off, as expected, is at $n_{ch} = 1$, and it is straightforward to see how equations (2)–(8) change.

Table 2. Results of fits to π^-p and π^-n data at $p_{lab} = 40$ GeV/c

Reaction	a	σ	a/σ	χ^2/N
π^-p	4·0	3·69	1·085	14/11
π^-n	3·6	3·6	1·0	12/9

4. Discussion

We have shown in the previous sections that the truncated gaussian adequately explains the pp data from 13 to 303 GeV/c. It can be seen from the values for a, σ and a/σ given in table 1 that over this energy range all three are increasing with energy. If this property continues indefinitely, then ultimately the truncation procedure will become insignificant (except for the description of the lowest topological cross section) and $P(x)$ will tend to the standard gaussian. In this case

$$\langle n_{ch}\rangle/D \rightarrow a/\sigma \underset{s\rightarrow\infty}{\rightarrow} \infty,$$

where $D = (\langle n_{ch}^2 - \langle n_{ch}\rangle^2\rangle)^{1/2}$ and s is the square of the centre-of-mass energy. The present behaviour (Jacob 1972) of $\langle n_{ch}\rangle/D$, and indeed the success of the Koba *et al* (1972) prediction (that $\langle n_{ch}\rangle P_{n_{ch}}$ depends only on $n_{ch}/\langle n_{ch}\rangle$) in the range 50–303 GeV/c, would then be only a passing phase of the data. If, however, a and σ increase but a/σ tends to a constant, we should indeed predict asymptotically both experimental features just quoted. Unfortunately, our a/σ can be varied considerably without producing a drastic increase in χ^2, so we are unable at present to decide between these two alternatives.

We have interpreted our phenomenological procedure within the context of a naive parton model. An alternative possibility, which raises a fundamental question, is whether we should consider the probability distribution in n (total number of particles produced) as more fundamental and hence simpler than that in n_{ch}. It is conceivable, although we have no example at present, that the anomalous weighting of the lowest charge modes and even the approximate gaussian structure are consequences of the reshuffling of the discrete cross section in changing from the distribution in the variable n to that in n_{ch}. This question requires further study and depends upon the correlations between the numbers of charged and neutral particles in the interaction.

In conclusion, we must emphasize that what we have presented in this paper is fundamentally a phenomenological procedure. Its range of validity and limitations should be further tested (when data become available) before a more detailed physical interpretation is warranted.

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